DERBY MOOR
SPENCER ACADEMY

Step up to Sixth Form: Mathematics Task 1

Part 1. Surds

1. (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$.

(2)

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where *n* is an integer.

(2)
(Total 4 marks)

2. Express each of the following in the form $m + n\sqrt{3}$, where *m* and *n* are integers:

(a) $(\sqrt{3}+1)^2$;

(2)

(b) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

(3)
(Total 5 marks)

3. (a) Express $(4\sqrt{5}-1)(\sqrt{5}+3)$ in the form $p + q\sqrt{5}$, where *p* and *q* are integers.

(3)

(b) Show that $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$ is an integer and find its value.

(3)
(Total 6 marks)

4. (a) Simplify $\left(\sqrt{12}+2\right)\left(\sqrt{12}-2\right)$.

(2)

(b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where *m* is an integer.

(1)

(c) Express $\dfrac{\sqrt{12}+2}{\sqrt{12}-2}$ in the form $a + b\sqrt{3}$, where *a* and *b* are integers.

(4)
(Total 7 marks)

5. Express $(4-\sqrt{7})(5+2\sqrt{7})$ in the form $a + b\sqrt{7}$, where *a* and *b* are integers.

(Total 3 marks)

6. (a) Express $(3+\sqrt{2})^2$ in the form $p + q\sqrt{2}$.

(2)

(b) Hence express $\dfrac{98}{(3+\sqrt{2})^2}$ in the form $m + n\sqrt{2}$, where *m* and *n* are integers.

(3)
(Total 5 marks)

Part 1. Surds – Answers

1.

(a) 1

(b) Answer = $5\sqrt{2}$

2.

(a) 4 + 2 $\sqrt{3}$

(b) 2 + $\sqrt{3}$

3.

a) $=17+11\sqrt{5}$

(b) 2

4.

(a) 8

(b) $2\sqrt{3}$

(c) $2+\sqrt{3}$

5. 6 + 3 $\sqrt{7}$

6.

(a) $=11+6\sqrt{2}$

(b) $=22-12\sqrt{12}$

Part 2. Completing the Square

Q1. (a) Factorise $x^2 - 4x - 12$.

(1)

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes.

(4)

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers.

(2)

(ii) Hence find the minimum value of $x^2 - 4x - 12$.

(1)

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer.

(2)

(Total 10 marks)

Q2. (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$.

(2)

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions.

(2)

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve.

(2)

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis.

(3)

(iii) Describe the geometrical transformation that maps the curve with equation

$y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$.

(3)

(Total 12 marks)

Q3. (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x + p)^2 + q$, where p and q are rational numbers.

(2)

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$.

(1)

(b) The point A has coordinates $(-3, 5)$ and the point B has coordinates $(x, 3x + 9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$.

(3)

(ii) Use your result from part (a)(ii) to find the minimum value of the length AB as x varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer.

(2)

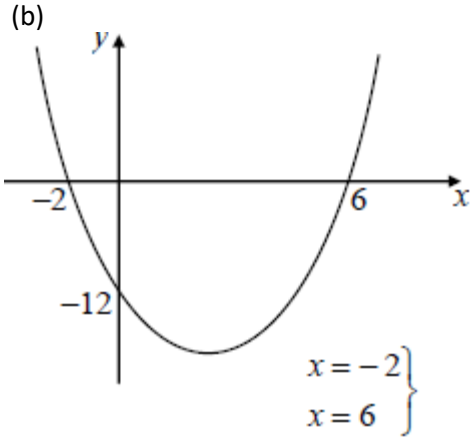
(Total 8 marks)

Part 2. Completing the Square – Answers

1.

(a) $(x - 6)(x + 2)$

(b)



c) (i) $(x - 2)^2 - 16$

(ii) (Minimum value is) –16

(d)

$y = [(x + 3)^2 - 4(x + 3) - 12] + 2$

or $y = (x + 1)^2 - 14$

or $y = x^2 + 2x - 13$

or $y - 2 = (x - 3)(x - 5)$

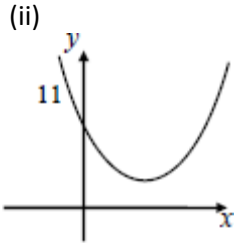
2.

(a) $(x - 3)^2 + 2$

(ii) $(x - 3)^2 = -2$ No (real) square root of −2 therefore equation has no real solutions

(b) (i) Vertex is at (3, 2)

(ii)



(iii) Translation through $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$

3.

(a) (i) $2(x + 1.5)^2 + 0.5$

(ii) (Minimum value is) 0.5

(b) (i) (AB^2 =) $(x + 3)^2 + (3x + 9 - 5)^2$ $\Rightarrow AB^2 = 5(2x^2 + 6x + 5)$

(ii) Minimum length of AB $= \frac{1}{2}\sqrt{n}$ with n = 10