



**DERBY MOOR
SPENCER ACADEMY**

Step up to Sixth Form: Mathematics Task 2



Part 1. Quadratic Formula and the Discriminant

Q1. The quadratic equation $(k + 1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geq 0$.

(3)

(b) Hence find the possible values of k .

(4)

(Total 7 marks)

Q2. The quadratic equation $(2k - 3)x^2 + 2x + (k - 1) = 0$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leq 0$.

(3)

(b)(i) Factorise $2k^2 - 5k + 2$.

(1)

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leq 0.$$

(3)

(Total 7 marks)

Q3. The quadratic equation

$$(2k - 7)x^2 - (k - 2)x + (k - 3) = 0$$

has real roots.

(a) Show that $7k^2 - 48k + 80 \leq 0$.

(4)

(b) Find the possible values of k .

(4)

(Total 8 marks)

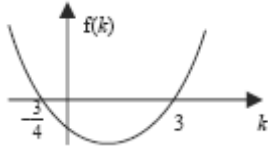
Part 2. Quadratic Formula and the Discriminant – Answers

1.

(a) $b^2 - 4ac = 16k^2 - 36(k + 1)$

$$\Rightarrow 4k^2 - 9k - 9 \geq 0$$

(b)



$$k \geq 3, k \leq -\frac{3}{4}$$

2.

(a) $b^2 - 4ac = 4 - 4(k - 1)(2k - 3)$

$$\Rightarrow 2k^2 - 5k + 2 \leq 0$$

(b) (i) $(2k - 1)(k - 2)$

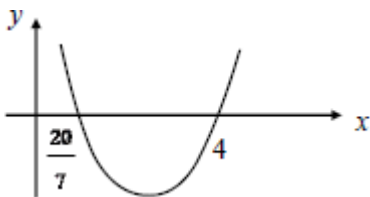
(ii) $0.5 \leq k \leq 2$

3.

(a) $(k - 2)^2 - 4 \times (2k - 7)(k - 3)$

$$7k^2 - 48k + 80 \leq 0$$

(b)



$$\frac{20}{7} \leq k \leq 4$$

Part 2. Equations and Circles

Q1. A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3)$$

(b) Write down:

(i) The coordinates of C ; (1)

(ii) The radius of the circle. (1)

(c) The point O has coordinates $(0,0)$.

(i) Find the length of CO . (2)

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. (2)

(Total 9 marks)

Q2. A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y -coordinates of the points where the circle crosses the y -axis. (3)

(b) Find the radius of the circle. (3)

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP , giving your answer in the form \sqrt{n} , where n is an integer. (2)

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ . (2)

(Total 10 marks)

Q3. A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$

(2)

(b) Write down:

(i) The coordinates of C ;

(1)

(ii) The radius of the circle, leaving your answer in surd form.

(1)

(c) A line has equation $y = 2x$.

(i) Show that the x -coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$.

(2)

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P .

(3)

(d) Prove that the point $Q(-1, 4)$ lies inside the circle.

(2)

(Total 11 marks)

Part 2. Equations and Circles – Answers

Q1.

(a) $(x - 4)^2 + (y + 3)^2$

(b) (i) Centre $(4, -3)$

(ii) Radius = 6

(c) (i) $CO = 5$

(ii) Consider CO and radius. $CO < r \Rightarrow O$ is **inside** the circle

Q2.

(a) $y = -2, 6$

(b) $r = \sqrt{25}$ or $r = \pm 5$ scores A0

(c) $\sqrt{34}$

(ii) $PQ = 3$

Q3.

(a) $x^2 + (y - 5)^2$ $k = 5$

(b) (i) Centre $(0, 5)$

(ii) Radius = $\sqrt{5}$

(c) (i) $x^2 + 4x^2 - 20x + 20 = 0$ *May substitute into original or "their (a)"* $x^2 - 4x + 4 = 0$

(ii) $(x - 2)^2 = 0$ or $x = 2$ Repeated root implies tangent

Point of contact is $P(2, 4)$

(d) $\sqrt{2} < \sqrt{5} \Rightarrow Q$ lies inside circle